# Opener

Let $f$ and $g$ be functions that are differentiable everywhere. If $g$ is the inverse function of $f$ and if $g(-2) = 5$ and $f'(5) = -\frac{1}{2}$, then $g'(-2) =$

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $-\frac{1}{5}$ (E) $-2$

# 3-7 Implicit Differentiation

**Learning Objectives:**

I can calculate the derivatives of implicitly defined function.

I can calculate the second and higher order derivatives of implicitly defined functions.

I can write the tangent and normal lines to implicitly defined functions.

**Implicit differentiation** is used whenever you need to find a rate of change (derivative) and the relation cannot be solved for y like with the equation:

$$x^3 y^2 - \cos y \cdot \ln x + e^x \sec^{-1} y = \sqrt{y^5 x^3}$$

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# Ex1. Find the derivative of

$$x^2+3x+2y+y^2=0$$

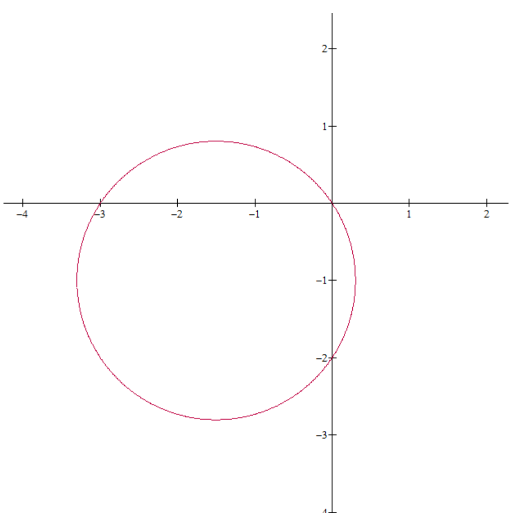
$$x^2+3x+2f(x)+(f(x))^2=0$$

$$2x+3+2\frac{dy}{dx}+2y\cdot\frac{dy}{dx}=0$$

$$2\frac{dy}{dx}+2y\frac{dy}{dx}=-2x-3$$

$$\frac{dy}{dx}(2+2y)=-2x-3$$

$$\boxed{\frac{dy}{dx}=\frac{-2x-3}{2+2y}}$$

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Ex3. Write the equation of the tangent line to the curve at the point (1,2)

f g $f' = 3$ $g' = \frac{dy}{dx}$

$$x^2 + 3xy + y^2 = 11$$

$$2x + 3y + 3x \cdot \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(3x+2y) = -2x-3y$$

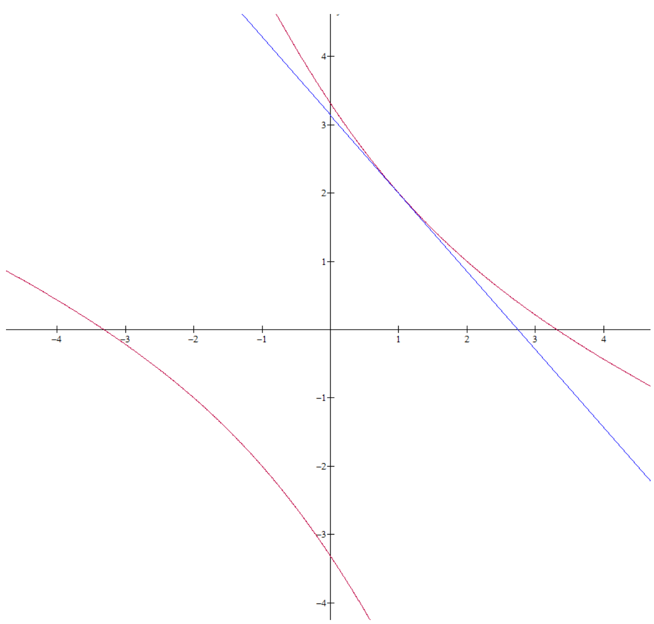
$$\frac{dy}{dx} = \frac{-2x-3y}{3x+2y}$$

$$\frac{dy}{dx} @ (1,2) = \frac{-2(1)-3(2)}{3(1)+2(2)}$$

$$= \frac{-2-6}{3+4} = \frac{-8}{7}$$

$$y - y_1 = m(x - x_1)$$

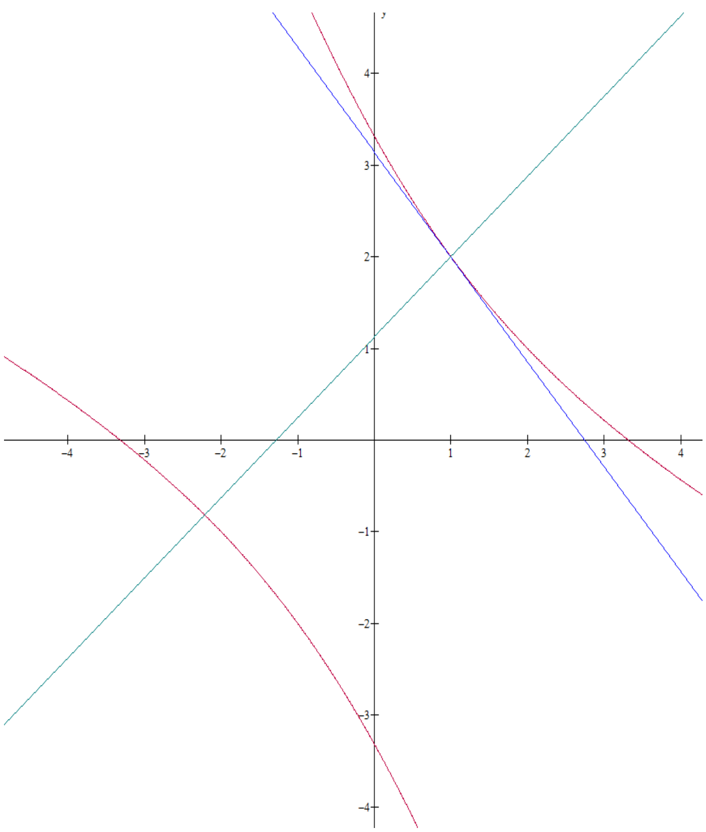
$$y - 2 = -8/7(x-1)$$



Ex4. Write the equation of the normal line to the curve at the point (1,2) (same curve that was in Ex3)

$$x^2 + 3xy + y^2 = 11$$

$$y - 2 = \frac{7}{8}(x - 1)$$



Ex5. Find the second derivative of

$$x^2+3x+2y+y^2=0$$

$$\frac{dy}{dx} = \frac{-2x-3}{2+2y} \quad f$$

$f' = -2$

$g' = 2\frac{dy}{dx}$

$+1(2x+3)$

$$\frac{d^2y}{dx^2} = \frac{-2\cdot(2+2y) - (-2x-3)\,2\frac{dy}{dx}}{(2+2y)^2}$$

$$= \frac{-2(2+2y) + 2(2x+3)\frac{-2x-3}{2+2y}}{(2+2y)^2}$$

$$= \frac{-2\cancel{(2+2y)}}{(2+2y)^{\cancel{2}}} + \frac{\frac{2(2x+3)(-2x-3)}{(2+2y)}}{(2+2y)^2}$$

$$\boxed{= \frac{-2}{2+2y} + \frac{2(2x+3)(-2x-3)}{(2+2y)^3}}$$

# Homework

pg 163 # 1-3, 8, 9, 12-14, 17, 22, 24, 28, 30, 43, 44, 47, 49, 54, 56, 61-64

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